Please check the examination details below before entering your candidate information

Candidate surname	Other names
WRITTEN MS	

Pearson Edexcel International GCSE

Centre Number | | | | | |

Candidate Number | | | | |

Thursday 5 November 2020

Morning (Time: 2 hours 30 minutes) | Paper Reference **4MB1/02**

Mathematics B

Paper 2

You must have:
Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- **Calculators may be used**.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.
- Without sufficient working, correct answers may be awarded no marks.

Turn over ▶

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1/1/1/1/

P 6 1 9 0 7 A 0 1 3 6

Pearson

Answer ALL TWELVE questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1

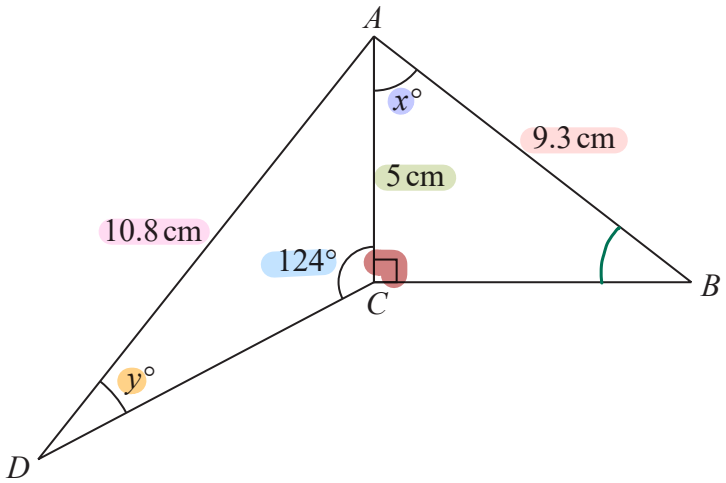


Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows a quadrilateral $ABCD$.

$AB = 9.3$ cm $AC = 5$ cm $AD = 10.8$ cm

$\angle ACD = 124°$ $\angle ACB = 90°$ $\angle CAB = x°$ $\angle ADC = y°$

Calculate to one decimal place,

(a) the value of x,

(2)

(b) the value of y.

(3)

(a) First, find $\angle ABC$ using sine rule.

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin ABC}{5} = \frac{\sin 90}{9.3}$

$\sin ABC = \frac{\sin 90}{9.3} \times 5$ (×5 on left, ×5 on right)

$ABC = \sin^{-1}\left(\frac{\sin 90}{9.3} \times 5\right)$ ($\sin^{-1}$ on left, $\sin^{-1}$ on right)

$ABC = 32.5227...$

Alternate method using SOHCAHTOA

$\cos x = \frac{5}{9.3}$

$x = \cos^{-1}\left(\frac{5}{9.3}\right)$

$= 57.477...$

$\approx 57.5°$ (1 d.p.)

[Sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$]

Question 1 continued

Angles in a triangle add to 180°.

$32.5227... + x + 90 = 180$

$x = 180 - 90 - 32.522..$

$x = 57.4772...$

$\approx 57.5°$ (1 d.p.)

(b) Use sine rule to find y.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{\sin y}{5} = \frac{\sin 124}{10.8}$$

$$\sin y = \frac{\sin 124}{10.8} \times 5$$

$$y = \sin^{-1}\left(\frac{\sin 124}{10.8} \times 5\right)$$

$= 22.5701...$

$\approx 22.6°$ (1 d.p.)

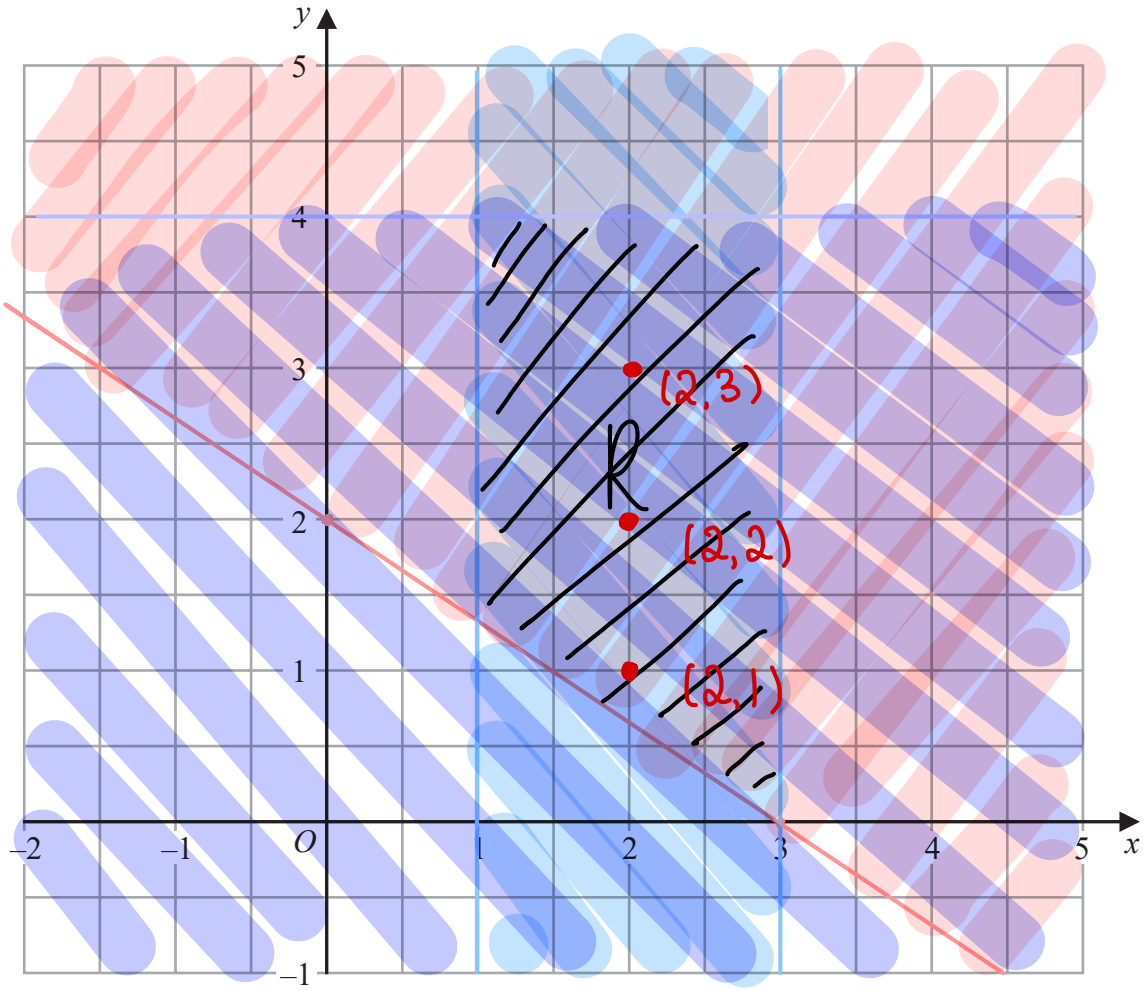
(Total for Question 1 is 5 marks)

2 (a) On the grid below, by drawing suitable straight lines and using shading, show the region **R** that satisfies the inequalities

$$2x + 3y \geqslant 6 \qquad 1 \leqslant x \leqslant 3 \qquad y \leqslant 4$$

Label the region **R**.

(3)



The point P with coordinates (x, y) lies in the region that satisfies the inequalities

$$2x + 3y > 6 \qquad 1 < x < 3 \qquad y < 4$$

Given that x and y are integers,

(b) write down the coordinates of all possible points P.

(2)

(a) Rearrange $2x+3y \geq 6$ in the form $y = mx + c$.

$2x+3y \geq 6$

$3y \geq 6-2x$

$y \geq 2 - \frac{2}{3}x$.

$y \geq -\frac{2}{3}x + 2$

Draw $y \geq -\frac{2}{3}x + 2$ which has y-intercept 2 and gradient $-\frac{2}{3}$. Since it is equal or greater than y, it is a solid line.

Question 2 continued

Draw two vertical solid lines at $x = 1$ and $x = 3$ and shade the region in between for $1 \leqslant x \leqslant 3$.

Draw a solid horizontal line at $y = 4$ for $y \leqslant 4$ and shade the region below the line.

Find the area where all shadings coincide and label this R.

(b) P can have any integer co-ordinate inside the region R.

P can be (2, 1)

(2, 2)

(2, 3)

(Total for Question 2 is 5 marks)

3 Express $(5x - 6) \div \left(\frac{25x^2 - 36}{x + 3}\right) - \frac{1}{x + 2}$ as a single fraction in its simplest form.

Show clear algebraic working.

(4)

Use BEDMAS to determine order of operations.

- Brackets
- Exponentials
- Division
- Multiplication
- Addition
- Subtraction

First tackle the division.

$$(5x - 6) \div \left(\frac{25x^2 - 36}{x + 3}\right)$$

Notice the difference of two squares

$25x^2 - 36 = (5x - 6)(5x + 6)$

$$= (5x-6) \div \left(\frac{(5x-6)(5x+6)}{x+3}\right)$$

$$= (5x-6) \times \frac{x+3}{(5x-6)(5x+6)}$$

Next complete the subtraction.

To subtract fractions, the denominators must be the same.

$$\frac{x+3}{5x+6} - \frac{1}{x+2} = \frac{(x+2)(x+3)}{(x+2)(5x+6)} - \frac{(5x+6)(1)}{(5x+6)(x+2)}$$

$$= \frac{(x+2)(x+3) - (5x+6)}{(5x+6)(x+2)}$$

Question 3 continued

$$= \frac{x^2 + 5x + 6 - (5x + 6)}{5x^2 + 16x + 12}$$

$$= \frac{x^2}{5x^2 + 16x + 12}$$

	x	2
x	x^2	$2x$
3	$3x$	6

$x^2 + 2x + 3x + 6$

$= x^2 + 5x + 6$

	$5x$	6
x	$5x^2$	$6x$
2	$10x$	12

$5x^2 + 6x + 10x + 12$

$= 5x^2 + 16x + 12$

(Total for Question 3 is 4 marks)

4

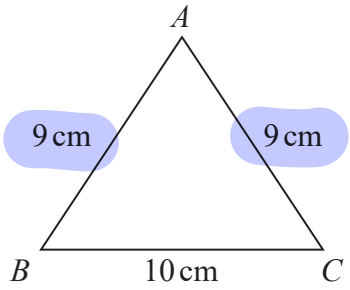


Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows triangle ABC.

$AB = 9\,\text{cm}$ $AC = 9\,\text{cm}$ $BC = 10\,\text{cm}$

(a) Calculate the area, in cm^2 to one decimal place, of triangle ABC.

(3)

Figure 3 shows two solid right prisms, each of length 7 cm.

One of the prisms has triangle ABC as a cross section of the prism.
The other prism has trapezium $DEFG$ as a cross section of the prism.

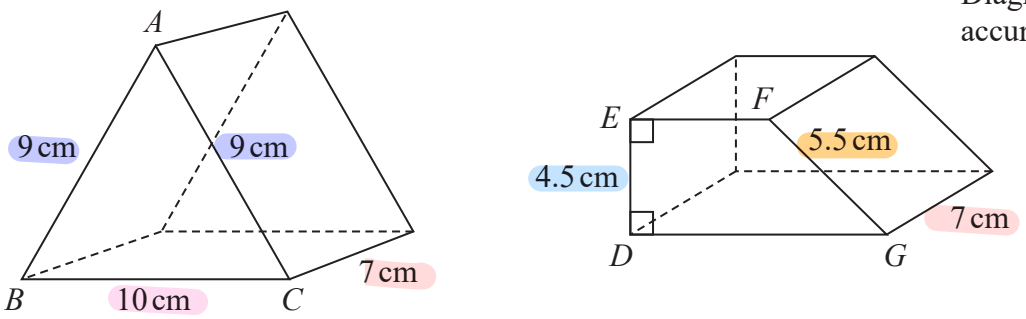


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Figure 3

In trapezium $DEFG$,

$DE = 4.5\,\text{cm}$ $FG = 5.5\,\text{cm}$

The perimeter of trapezium $DEFG$ is equal to the perimeter of triangle ABC.

(b) Calculate the difference, in cm^3 to one decimal place, between the volumes of the two prisms.

(5)

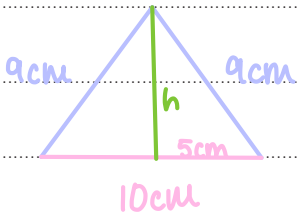
(a) Area of a triangle = $\frac{1}{2}$ × base × height

You can find the height using Pythagoras

$a^2 + b^2 = c^2$

$$\left[\text{Area of trapezium} = \frac{1}{2}(a+b)h\right]$$

Question 4 continued



$h^2 + 5^2 = 9^2$

$h^2 = 9^2 - 5^2$

$h = \sqrt{9^2 - 5^2}$

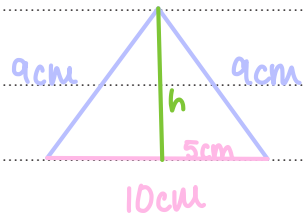
$= 7.4833...$

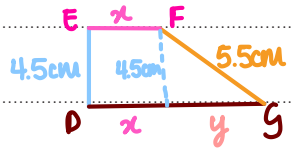
Area ABC $= \frac{1}{2} \times 10 \times 7.4833...$

$= 37.41...$

$\approx 37.4\ cm^2$ (1 d.p.)

(b) Use geometry and the fact that the perimeter of ABC is equal to the perimeter of DEFG to find EF and DG.





Perimeter ABC $= 9 + 9 + 10$

$= 28cm$

Perimeter of DEFG $= 4.5 + 5.5 + x + x + 3.16...$

$= 13.16... + 2x$

$13.16... + 2x = 28$

$2x = 14.82...$

$x = 7.418...$

EF $= 7.418...$ cm

You can use Pythagoras to find y

$4.5^2 + y^2 = 5.5^2$

$y^2 = 5.5^2 - 4.5^2$

$y = \sqrt{5.5^2 - 4.5^2}$

$= 3.1622...$

DG $= 7.418... + 3.162... = 10.58...$

Question 4 continued

Volume of a prism = Cross-sectional area × length

Volume ABC = 37.41... × 7

= 261.91...

Area trapezium DEFG = $\frac{1}{2}(7.418... + 10.58...) \times 4.5$

= 40.5

Volume DEFG prism = 40.5 × 7

= 283.5

Difference in volume = 283.5 − 261.91...

= 21.583...

≈ 21.6 cm³ (1 d.p.)

Question 4 continued

(Total for Question 4 is 8 marks)

5 Tahina travels to work by bus.
Her total bus fare last week was £12.50
This week her total bus fare has increased by 8%

(a) Calculate her total bus fare for this week.

(2)

Tahina works in a kiosk selling hot drinks.
She sells coffee, tea and hot chocolate.

On Monday, Tahina sold a total of 378 hot drinks.
The numbers of cups of coffee, tea and hot chocolate she sold were in the ratios 5 : 3 : 1

(b) Calculate the difference between the number of cups of coffee and the number of cups of hot chocolate that Tahina sold on Monday.

(3)

On Monday, $\frac{3}{14}$ of the number of cups of coffee Tahina sold were sold without milk.

(c) Calculate the number of cups of coffee that Tahina sold without milk.

(2)

The cost of each cup of coffee that Tahina sells from the kiosk is £2.80

Tahina went on holiday to the USA and to Canada.
She bought a cup of coffee in the USA for $3.20

Using an exchange rate of £1 = $1.24

(d) compare the cost of each cup of coffee sold from Tahina's kiosk with the cost of the cup of coffee that Tahina bought in the USA.

(2)

In Canada, Tahina bought a sandwich for 5.28 Canadian dollars.

Using exchange rates of

£1 = $1.24 and 1 Canadian dollar = $0.75

(e) convert 5.28 Canadian dollars to pounds (£)
Give your answer to 2 decimal places.

(3)

(a) There are two main ways to do this:

1. Multiplier (faster)

Since her weekly bus fare increases by 8%, she will be 108% of original.

$12.50 \times \frac{108}{100} = £13.50$

2. Finding percentages

Find 8% of £12.50

$12.50 \times \frac{8}{100} = 1$

Add 8% onto £12.50

$12.50 + 1 = £13.50$

Question 5 continued

(b) Find the multiplier:

$378 \div (5+3+1) = 378 \div 9$

$= 42$

	Coffee	:	Tea	:	Hot Chocolate
	5	:	3	:	1
×42	210	:	126	:	42

(×42 from 5 : 3 : 1 to 210 : 126 : 42; ×42 from 1 to 42)

Find difference:

$210 - 42 = 168$

(c) Find $\frac{3}{14}$ of 210.

$210 \times \frac{3}{14} = 45$

(d) Cost of cup of coffee from Tahira's kiosk in pounds: £2.80

Cost of cup of coffee from USA in pounds: $3.20 \div 1.24 = 2.58$

$2.80 - 2.58 =$ £0.22

The coffee is more expensive from the kiosk.

(e) First convert Canadian dollars to American dollars.

$5.28 \times 0.75 = 3.96$

Then convert American dollars to pounds.

$3.96 \div 1.24 = 3.1935...$

$\approx$ £3.19 (2 d.p.)

Question 5 continued

Question 5 continued

(Total for Question 5 is 12 marks)

6 Solve the simultaneous equations

$$x^2 + y^2 = 26$$
$$2x + y = 9$$

Show clear algebraic working.

(6)

Rearrange second equation to make y the subject.

$2x + y = 9$

$-2x$ $-2x$

$y = 9 - 2x$

Substitute y into the first equation.

$x^2 + (9-2x)^2 = 26$

Expand → $x^2 + (4x^2 - 36x + 81) = 26$

Simplify → $5x^2 - 36x + 81 = 26$

-26 -26

$5x^2 - 36x + 55 = 0$

	9	$-2x$
9	81	$-18x$
$-2x$	$-18x$	$4x^2$

$81 - 18x - 18x + 4x^2$

$= 4x^2 - 36x + 81$

What two numbers add to make -36 and multiply to make 275?

-25 and -11

$5 \times 55 = 275$ (−5)

$(5x - 25)(5x - 11) = 0$

$(x - 5)(5x - 11) = 0$

$x - 5 = 0$ $+5$ $+5$

$x = 5$

$5x - 11 = 0$ $+11$ $+11$

$5x = 11$ $\div 5$ $\div 5$

$x = \frac{11}{5}$

Substitute values of x into second equation to find y values.

$y = 9 - 2(5)$

$y = 9 - 10$

$y = -1$

$y = 9 - 2\left(\frac{11}{5}\right)$

$y = 9 - \frac{22}{5}$

$y = \frac{23}{5}$

$\therefore$ $x = 5$ $y = -1$ and $x = \frac{11}{5}$ $y = \frac{23}{5}$

Question 6 continued

(Total for Question 6 is 6 marks)

7

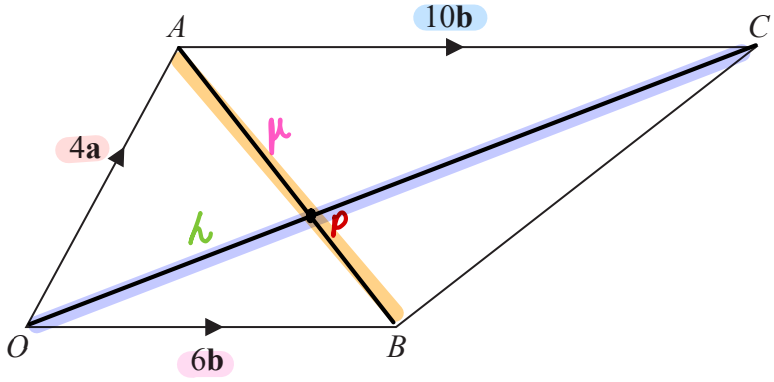


Diagram **NOT** accurately drawn

Figure 4

Figure 4 shows a trapezium $OACB$.

$\overrightarrow{OA} = 4\mathbf{a}$ $\overrightarrow{OB} = 6\mathbf{b}$ $\overrightarrow{AC} = 10\mathbf{b}$

P is the point on AB such that OPC is a straight line.

Find an expression for $\overrightarrow{PC}$ in terms of **a** and **b**.
Give your answer in its simplest form.

(6)

Since we do not know the ratios for $\overrightarrow{AP} : \overrightarrow{PB}$ and $\overrightarrow{OP} : \overrightarrow{PC}$, we can assign

μ for $\overrightarrow{AP}$ and λ for $\overrightarrow{OP}$

$\overrightarrow{AP} = \mu\overrightarrow{AB}$

$\overrightarrow{AB} = -\overrightarrow{AO} + \overrightarrow{OB}$

$\overrightarrow{AB} = -4a + 6b$

$\overrightarrow{AP} = \mu(6b - 4a)$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$

$\overrightarrow{OC} = 4a + 10b$

$\overrightarrow{OP} = \lambda(4a + 10b)$

$\overrightarrow{AP} = -\overrightarrow{AO} + \overrightarrow{OP}$

$\overrightarrow{AP} = -4a + \lambda(4a + 10b)$

Question 7 continued

$\mu(6b - 4a) = -4a + \lambda(4a + 10b)$

Separate in terms of a and b:

$\mu(6b) = \lambda(10b)$ $\quad$ $\mu(-4a) = -4a + \lambda(4\mu)$

$6\mu = 10\lambda$ $\quad$ $-4\mu = -4 + 4\lambda$

$\mu = \frac{10\lambda}{6}$

$-4\left(\frac{10\lambda}{6}\right) = -4 + 4\lambda$

$\frac{-40\lambda}{6} = -4 + 4\lambda$

$-40\lambda = -24 + 24\lambda$

$0 = -24 + 64\lambda$

$24 = 64\lambda$

$\frac{24}{64} = \lambda$

$\frac{3}{8} = \lambda$

Since $\vec{OP}$ is $\frac{3}{8}$ of $\vec{OC}$, $\vec{PC}$ must be $\frac{5}{8}$ of $\vec{OC}$.

$\vec{PC} = \frac{5}{8}(4a + 10b)$

$= \frac{20}{8}a + \frac{50}{8}b$

$= \frac{5}{2}a + \frac{25}{4}b$

(Total for Question 7 is 6 marks)

8 The diagram shows kite A.

Kite B is the image of kite A under a reflection in the line with equation $y = -1$

(a) On the grid opposite, draw and label kite B.

(2)

Kite A is transformed to kite C by a reflection in the line with equation $x = 4$ followed by the translation $\begin{pmatrix} -8 \\ -6 \end{pmatrix}$

Kite C is the image of kite B under a rotation through 180° about the point P.

(b) (i) On the grid opposite, draw and label kite C.

(ii) Find the coordinates of P.

(3)

$$\mathbf{M} = \begin{pmatrix} 0 & -1 \\ -2 & 0 \end{pmatrix} \quad \mathbf{N} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Kite A is transformed to kite D under the combined transformation with matrix **MN**.

(c) On the grid opposite, draw and label kite D.

(4)

Given that $\dfrac{\text{area of kite } D}{\text{area of kite } A} = k$

(d) find the value of k.

(1)

(a) Draw a horizontal line $y=-1$ and reflect kite A.

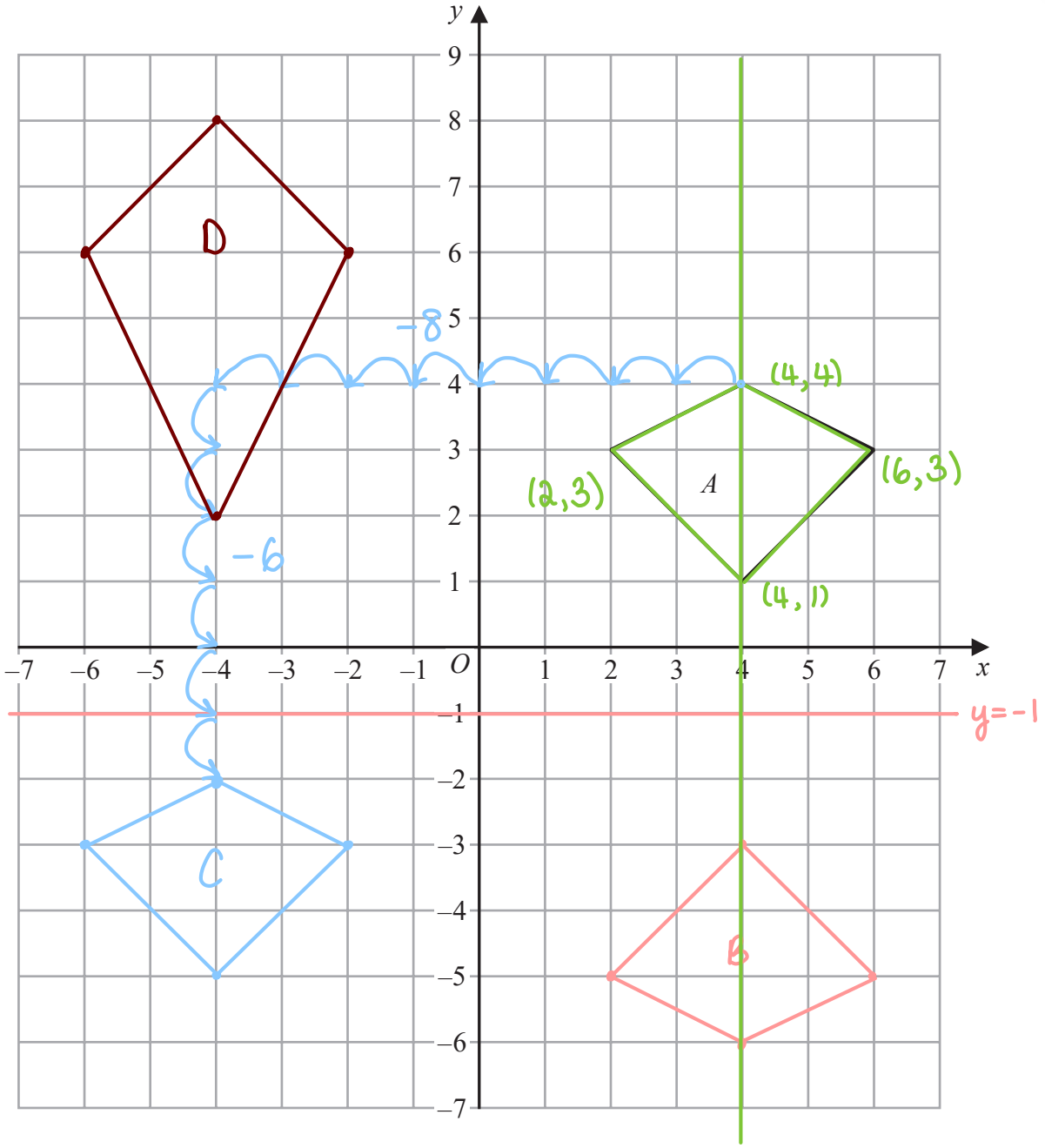
(b) (i) Draw vertical line $x=4$ and reflect kite A.

Translate kite using vector $\begin{pmatrix} -8 \\ -6 \end{pmatrix}$

(ii) Using tracing paper, rotate kite B 180° and find co-ordinates of P.

P has co-ordinates

Question 8 continued



Turn over for a spare grid if you need to redraw your kites.

Question 8 continued

(c) First multiply matrices M and N to find MN.

Multiplying 2×2 Matrices:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -2 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} (0)(0)+(-1)(1) & (0)(-1)+(-1)(0) \\ (-2)(0)+(0)(1) & (-2)(-1)+(0)(0) \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$$

Multiplying 2×2 by 2×1 matrices:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax+by \\ cx+dy \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 4 \end{pmatrix} = \begin{pmatrix} (-1)(4)+(0)(4) \\ (0)(4)+(2)(4) \end{pmatrix}$$

$$= \begin{pmatrix} -4+0 \\ 0+8 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ 8 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} (-1)(2)+(0)(3) \\ (0)(2)+(2)(3) \end{pmatrix}$$

$$= \begin{pmatrix} -2+0 \\ 0+6 \end{pmatrix}$$

$$= \begin{pmatrix} -2 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 6 \\ 3 \end{pmatrix} = \begin{pmatrix} (-1)(6)+(0)(3) \\ (0)(6)+(2)(3) \end{pmatrix}$$

$$= \begin{pmatrix} -6+0 \\ 0+6 \end{pmatrix}$$

$$= \begin{pmatrix} -6 \\ 6 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} (-1)(4)+(0)(1) \\ (0)(4)+(2)(1) \end{pmatrix}$$

$$= \begin{pmatrix} -4+0 \\ 0+2 \end{pmatrix}$$

$$= \begin{pmatrix} -4 \\ 2 \end{pmatrix}$$

Question 8 continued

(d) Find the areas of each kite by dividing them into two triangles.

Area of a triangle = $\frac{1}{2}$ × base × height

Kite A = $\left[\frac{1}{2} \times 1 \times 4\right] + \left[\frac{1}{2} \times 2 \times 4\right]$

$= [2] + [4]$

$= 6$

Kite D = $\left[\frac{1}{2} \times 2 \times 4\right] + \left[\frac{1}{2} \times 4 \times 4\right]$

$= [4] + [8]$

$= 12$

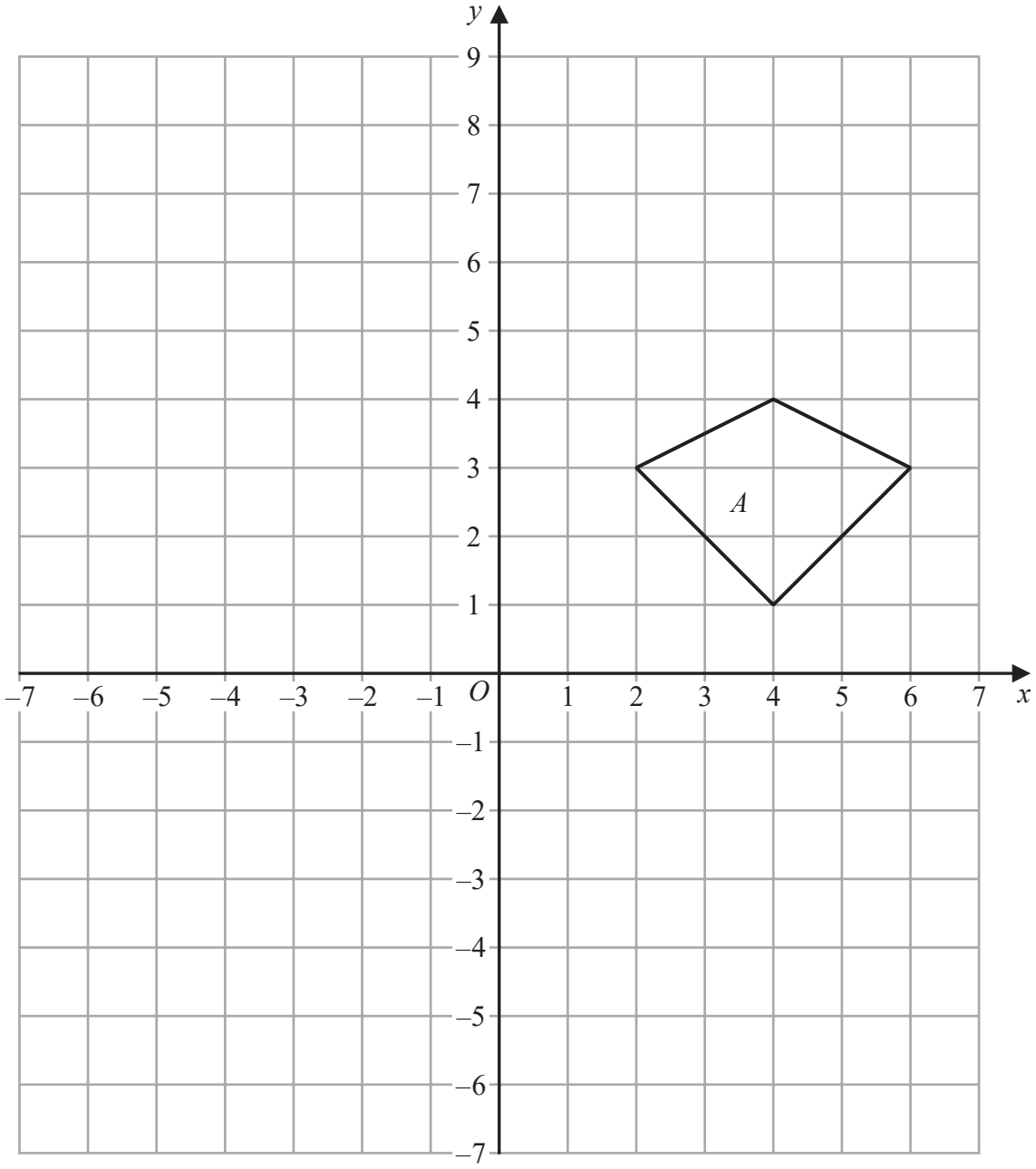
$\frac{\text{Area of Kite D}}{\text{Area of Kite A}} = k.$

$\frac{12}{6} = 2$

$k = 2$

Question 8 continued

Only use this grid if you need to redraw your kites.



(Total for Question 8 is 10 marks)

9 90 students from a sixth form college were each asked the following question.

"Do you study any of Biology (B), Chemistry (C) or Physics (P)?"

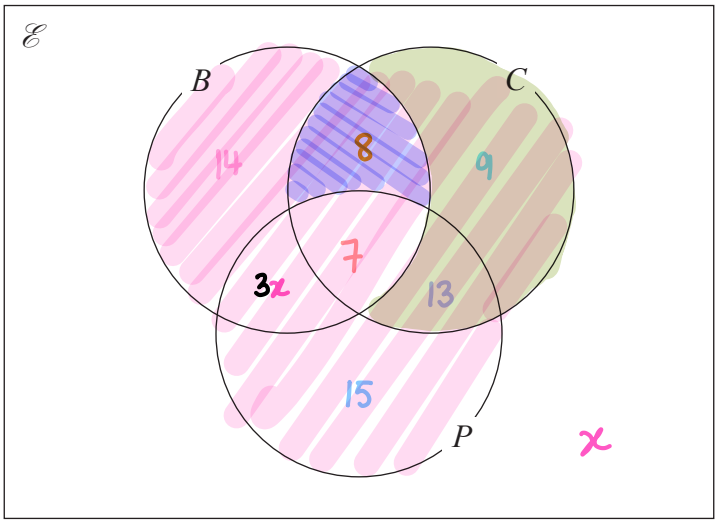
Of these 90 students

7 study all three subjects
15 study Biology and Chemistry
20 study Chemistry and Physics
37 study Chemistry
14 study Biology only
15 study Physics only

The number of these students who study Biology and Physics but not Chemistry is three times the number of these students who study none of these three subjects.

Let x be the number of these students who study none of these three subjects.

(a) Show all this information on the Venn diagram, giving the number of students in each appropriate subset, in terms of x where necessary.



(3)

(b) Find the value of x.

(2)

(c) Find

(i) $n(B' \cap C)$

(ii) $n(B \cup C \cup P)$

(iii) $n(B \cap C \cap P')$

(3)

One of the students is to be chosen at random.
Given that this student studies Chemistry,

(d) find the probability that this student also studies Physics.

(2)

Question 9 continued

(a) $15 - 7 = 8$

$20 - 7 = 13$

$37 - 8 - 7 - 13 = 9$

(b) $14 + 8 + 7 + 9 + 13 + 15 + 3x + x = 90$

$66 + 4x = 90$

-66 $\quad -66$

$4x = 24$

$\div 4$ $\quad \div 4$

$x = 6$

(c) (i) $n(B' \cap C)$ means the number of students who study Chemistry but not Biology.

$n(B' \cap C) = 9 + 13$

$= 22$

(ii) $n(B \cup C \cup P)$ means the number of students who study either Biology, Chemistry or Physics.

$n(B \cup C \cup P) = 14 + 8 + 9 + 7 + 13 + 15 + 3(6)$

$= 84$

(iii) $n(B \cap C \cap P')$ means the number of students who study both Biology and Chemistry but not Physics.

$n(B \cap C \cap P') = 8$

(Total for Question 9 is 10 marks)

Question 9 continued

(d) This means the probability that a student studies Physics given that they study Chemistry.

$$\frac{n(C \cap P)}{n(C)} = \frac{7 + 13}{8 + 9 + 7 + 13}$$

$$= \frac{20}{37}$$

(Total for Question 9 is 10 marks)

10 The function g is such that

$$g: x \mapsto \frac{12}{x-3}$$

(a) State the value of x that must be excluded from any domain of g.

(1)

The function h is defined for all values of x by

$$h: x \mapsto x^2 + 4$$

(b) Write down the range of h.

(1)

(c) Find h(–9)

(1)

(d) Find hg(5)

(2)

(e) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto \ldots$

(2)

(f) Show that hg(x) can be written as $\frac{4(45 - 6x + x^2)}{(x-3)^2}$

(4)

(g) Solve hg(x) = 5
Show clear algebraic working.

(3)

(a) Since dividing any number by 0 is undefined, any values of x for which $x-3=0$ must be excluded.

$x - 3 = 0$

+3 ... +3

$x = 3$

(b) The range is all possible outcomes.

Since any number squared is positive the smallest number x^2 can be is 0, and any value of x^2 must have 4 added to it.

∴ Range of $h: x$ is $h: x \geqslant 4$

Question 10 continued

(c) Substitute $x = -9$ into $h:x$ to find $h(-9)$.

$h(-9) = (-9)^2 + 4$

$= 81 + 4$

$= 85$

(d) Substitute $x = 5$ into $g(x)$ and then substitute this into $h(x)$.

$g(5) = \frac{12}{(5) - 3}$

$= \frac{12}{2}$

$= 6$

$h(6) = (6)^2 + 4$

$= 36 + 4$

$= 40$

$\therefore hg(5) = 40$

(e) Swap $g:x$ for y and rearrange to make x the subject.

$y = \frac{12}{x - 3}$

$\times (x - 3)$ $\qquad$ $\times (x - 3)$

$y(x - 3) = 12$

Expand brackets

$yx - 3y = 12$

$+3y$ $\qquad$ $+3y$

$yx = 12 + 3y$

$\div y$ $\qquad$ $\div y$

$x = \frac{12 + 3y}{y}$

$g^{-1}: x \to \frac{12+3y}{y}$

(f) Substitute g(x) into h(x) for hg(x).

$hg(x) = \left(\frac{12}{x-3}\right)^2 + 4$

$= \left(\frac{12^2}{(x-3)^2}\right) + \frac{4(x-3)^2}{(x-3)^2}$

$= \frac{144}{(x-3)^2} + \frac{4(x^2-6x+9)}{(x-3)^2}$

$= \frac{144 + 4x^2 - 24x + 36}{(x-3)^2}$

$= \frac{4x^2 - 24x + 180}{(x-3)^2}$

$= \frac{4(x^2 - 6x + 45)}{(x-3)^2}$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$x^2 - 3x - 3x + 9$

$= x^2 - 6x + 9$

(e) $\frac{4(x^2 - 6x + 45)}{(x-3)^2} = 5$

$\times (x-3)^2$ $\qquad$ $\times (x-3)^2$

$4(x^2 - 6x + 45) = 5(x^2 - 6x + 9)$

$4x^2 - 24x + 180 = 5x^2 - 30x + 45$

$-4x^2$ $\qquad$ $-4x^2$

$-24x + 180 = x^2 - 30x + 45$

$+24x$ $\qquad$ $+24x$

$180 = x^2 - 6x + 45$

-180 $\qquad$ -180

$0 = x^2 - 6x - 135$

$0 = (x - 15)(x + 9)$

What two numbers add to make -6 and multiply to make -135? 9 and -15

Question 10 continued

$x - 15 = 0$ $\quad +15 \quad +15$

$x = 15$

$x + 9 = 0$ $\quad -9 \quad -9$

$x = -9$

(Total for Question 10 is 14 marks)

11 (a) Complete the table of values for $y = 2x^3 - 3x + 4$

x	–2.5	–2	–1.5	–1	–0.5	0	0.5	1	1.5	2	2.5
y	–19.75	–6	1.75	5	5.25	4	2.75	3	6.25	14	27.75

(3)

(b) On the grid, plot the points from your completed table and join them to form a smooth curve.

(3)

(c) By drawing on the grid a suitable tangent to the curve, find an estimate, to the nearest whole number, of the gradient of the curve at the point where $x = 2$

(2)

(d) Use your graph to find the range of values of x, in $-2.5 \leqslant x \leqslant 2.5$, for which $2x^3 - 3x + 4 > 5x$

(2)

(a) Complete table by substituting values of x into the equation $y = 2x^3 - 3x + 4$

$y = 2(-2)^3 - 3(-2) + 4$

$y = -6$

$y = 2(-1)^3 - 3(-1) + 4$

$y = 5$

$y = 2(0)^3 - 3(0) + 4$

$y = 4$

$y = 2(1)^3 - 3(1) + 4$

$y = 3$

(b) Plot points and draw smooth curve going through all points

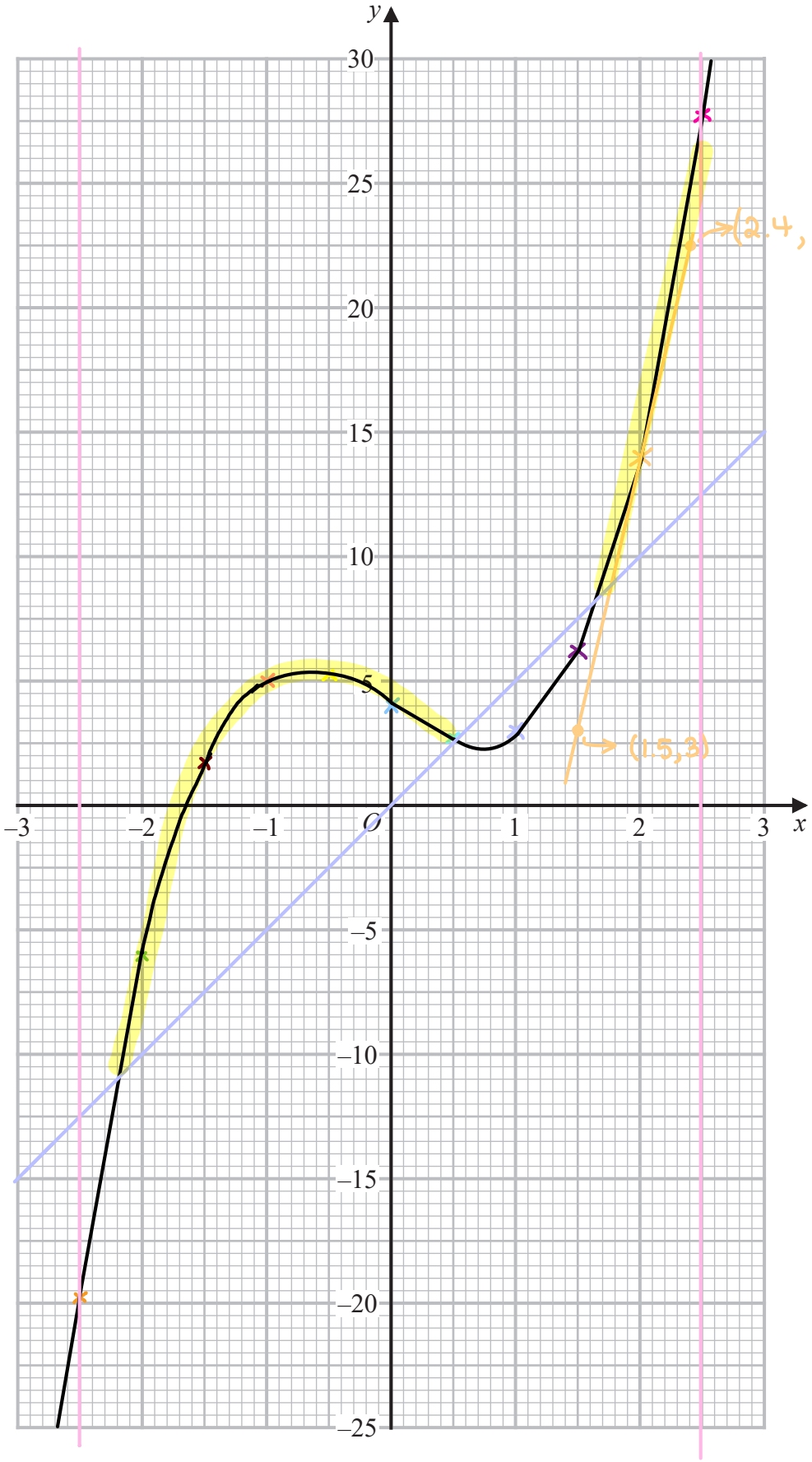
(c) Draw a straight line tangent and $x = 2$ and find gradient using two points

Gradient $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$\dfrac{22.5 - 3}{2.4 - 1.5} = 21.666\ldots$

≈ 21.7

Question 11 continued



Turn over for a spare grid if you need to redraw your graph.

Question 11 continued

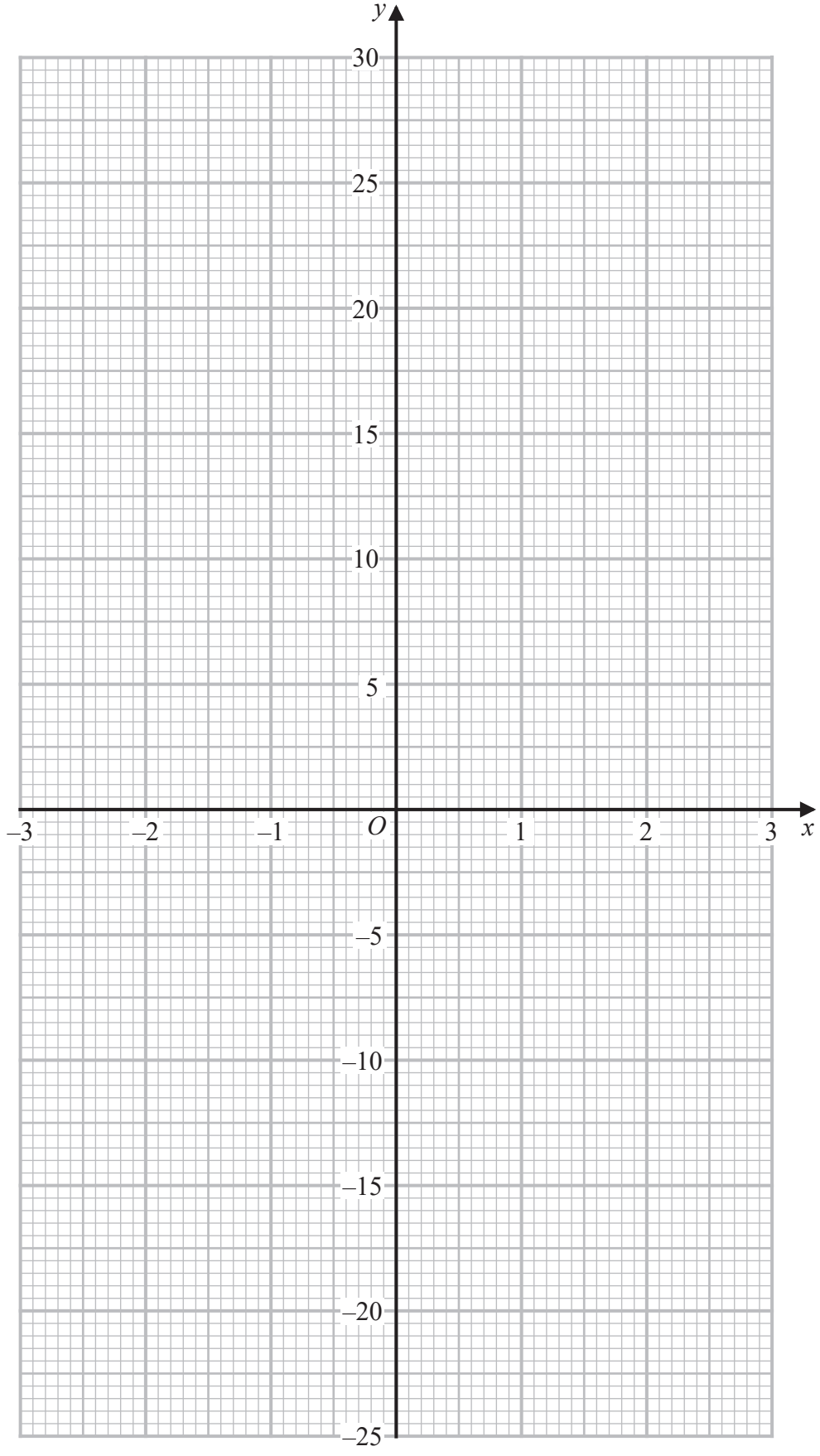
(d) Draw a straight diagonal line $y = 5x$ and draw vertical lines for the region $-2.5 \leq x \leq 2.5$

The regions where $2x^3 - 3x + 4$ is bigger than $5x$ is shaded yellow.

These are $-2.2 < x < 0.5$ and $1.7 < x \leq 2.5$

Question 11 continued

Only use this grid if you need to redraw your graph.



(Total for Question 11 is 10 marks)

12 Harvey has two bags of counters, bag A and bag B.

Bag A contains 8 red counters, 3 orange counters and 4 yellow counters only.
Bag B contains 3 red counters, 2 orange counters and 5 green counters only.

Harvey takes at random one counter from bag A and one counter from bag B.

(a) Complete the probability tree diagram on the opposite page.

(3)

(b) Calculate the probability that the two counters taken have different colours.

(3)

Harvey replaces the counters into the bags from which he took the counters.

Chen has a bag, bag C, that contains 10 red counters and x orange counters only.

Chen takes at random one counter from bag A, one counter from bag B and one counter from bag C.

The probability that all three counters that Chen takes have the same colour is $\frac{1}{15}$

(c) Find the value of x.

(4)

(b) There are two ways to do this:

1. Add all probabilities for two different colours.

The possible combinations are: RO RG OR OG YR YO YG

$$\left(\frac{8}{15}\times\frac{2}{10}\right)+\left(\frac{8}{15}\times\frac{5}{10}\right)+\left(\frac{3}{15}\times\frac{3}{10}\right)+\left(\frac{3}{15}\times\frac{5}{10}\right)+\left(\frac{4}{15}\times\frac{3}{10}\right)+\left(\frac{4}{15}\times\frac{2}{10}\right)+\left(\frac{4}{15}\times\frac{5}{10}\right)$$

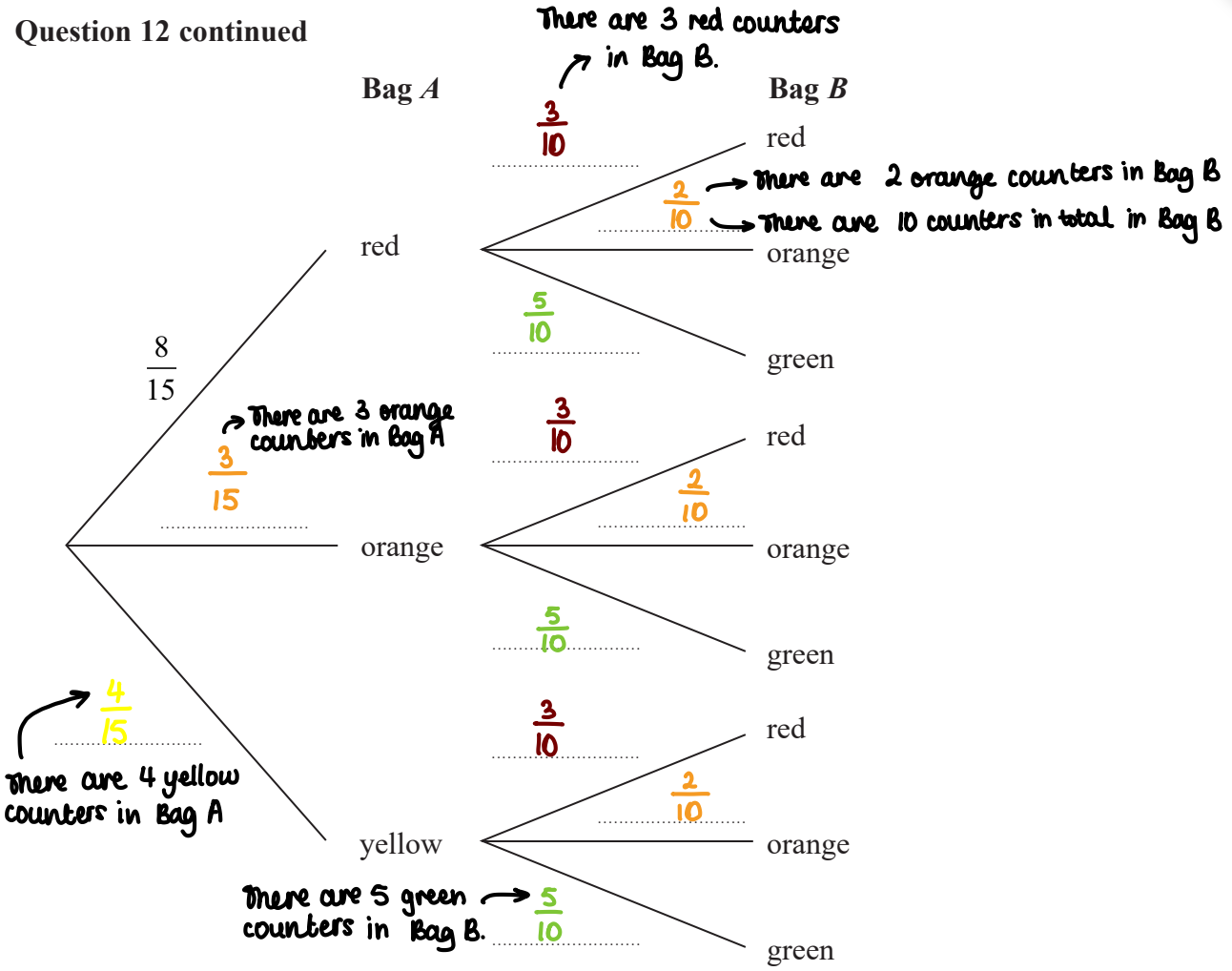
$$=\frac{4}{5}$$

2. Subtract probabilities of two counters of the same colour from 1.

Same colour combinations: RR OO

$$1-\left[\left(\frac{8}{15}\times\frac{3}{10}\right)+\left(\frac{3}{15}\times\frac{2}{10}\right)\right]=\frac{4}{5}$$

Question 12 continued



(c) The probability that a red counter is chosen from Bag C is $\frac{10}{10+x}$

The probability that an orange counter is chosen from Bag C is $\frac{x}{10+x}$

There are two possible combinations for all three counters being the same colour:

RRR or OOO

$$\left(\frac{8}{15} \times \frac{3}{10} \times \frac{10}{10+x}\right) + \left(\frac{3}{15} \times \frac{2}{10} \times \frac{x}{10+x}\right) = \frac{1}{15}$$

$$\left(\frac{24}{150} \times \frac{10}{(10+x)}\right) + \left(\frac{6}{150} \times \frac{x}{(10+x)}\right) = \frac{1}{15}$$

$$\left(\frac{24x}{1500+150x}\right) + \left(\frac{6x}{1500+150x}\right) = \frac{1}{15}$$

Question 12 continued

$$\frac{30x}{1500+150x} = \frac{1}{15}$$

$\times (1500+150x)$ $\qquad$ $\times (1500+150x)$

$$30x = \frac{1500+150x}{15}$$

$\times 15$ $\qquad$ $\times 15$

$$450x = 1500+150x$$

-1500 $\qquad$ -1500

$$300x = 1500$$

$\div 300$ $\qquad$ $\div 300$

$$x = 5$$

(Total for Question 12 is 10 marks)

TOTAL FOR PAPER IS 100 MARKS